# Two-Factor Mixed and Within-Participants Designs

PSYC214: Statistics For Group Comparisons

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Week 8

# Learning Objectives

- Two-factor mixed and within-participants designs
- Focus on procedures rather than how the analysis is calculated
- How to interpret ANOVA tables and graphs
- Handling significant main effects, and simple main effects, of factors with three or more levels

# Introduction

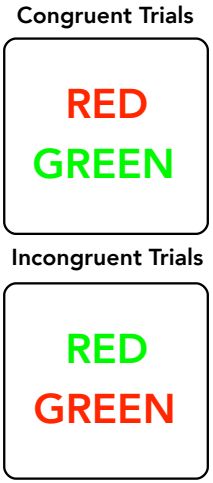
- We have now covered the three most mathematically straightforward designs:
  1. splitting total variability into between-group variability and within-group variability — one factor between-participants designs
  2. splitting within-group variability into between-participant variability and residual variability — one factor within-participants designs
  3. splitting between-group variability into main effect and interaction variability — $2 \times 2$ between-participants design
- There is little new to learn from analysing more complicated designs by hand

# A Two-Factor Mixed Design

- Mixed design ANOVAs are particularly versatile and are often used in psychology
- These designs have at least one between-participants factor and at least one within-participants factor
- Allows the researcher to capitalise on the benefits of between- and within-participants designs within the same design
- Let's consider an example based on the Stroop task

# The Stroop Task

- In the Stroop task, participants must name the ink colour of a colour word as quickly as possible:
  - on congruent trials, the ink colour and colour name are consistent
  - on incongruent trials, the ink colour and colour name are inconsistent
- Stroop effect = longer RTs for incongruent, compared to congruent, trials
- A measure of **response inhibition**



# Example of A Mixed Design

- A researcher wants to know if response inhibition is impaired in patients with Schizophrenia using the Stroop task
- She employs a $2 \times 2$ mixed design:
  - patient group: healthy *vs.* schizophrenia
  - trial type: congruent *vs.* incongruent
- patient group is necessarily a between-participants factor
- trial type is a within-participants factor
- There are $2 \times 2 = 4$ conditions; two groups of participants (healthy *vs.* schizophrenia) each complete two conditions of the experiment (congruent *vs.* incongruent trials)

# Hypothetical Data For Mixed-Design Stroop Experiment

| | | | *Factor B: Trial type (within participants)* | |
|---|---|---|---|---|
| | | | *Level $B_1$* | *Level $B_2$* |
| | | | *congruent* | *incongruent* |
| Factor *A*: | Level $A_1$ healthy | $P_1$ | 680 | 790 |
| Group (between participants) | | $P_2$ | 616 | 746 |
| | | $P_3$ | 530 | 670 |
| | | $P_4$ | 630 | 830 |
| | | $P_5$ | 694 | 794 |
| | Level $A_2$ schizophrenia | $P_6$ | 630 | 852 |
| | | $P_7$ | 610 | 875 |
| | | $P_8$ | 602 | 863 |
| | | $P_9$ | 660 | 912 |
| | | $P_{10}$ | 673 | 928 |

# Hypothetical Data For Mixed Design-Stroop Experiment

| | | *Factor B: Trial type* | | |
|---|---|---|---|---|
| | | *Level $B_1$* *congruent* | *Level $B_2$* *incongruent* | Overall |
| Factor *A*: Group | Level $A_1$ healthy | 630 | 776 | 703 |
| | Level $A_2$ schizophrenia | 635 | 885 | 760 |
| | Overall | 632.5 | 830.5 | |

# Error Terms In A Mixed-Design ANOA

- Recall that a between-participants design uses the within-group variance as its error term
- By contrast, a within-participants design uses the residual variance as its error term
- A mixed-design ANOVA produces two error terms:
  1. one for the between-participants main effect
  2. one for the within-participants main effect and the interaction
- You must be careful to ensure when reporting the ANOVA that the correct degrees of freedom are read from the table

# ANOVA Table For Mixed-Design Stroop Experiment

| Source | Sum of Squares | Degrees of Freedom | Mean Square | $F$ | $P$ |
|---|---|---|---|---|---|
| $A$ (group) | 19531.250 | 1 | 19531.250 | 4.307 | 0.072 |
| Error S/A (Bet-ss) | 36281.000 | 8 | 4535.125 | | |
| | | | | | |
| $B$ (trial type) | 187211.250 | 1 | 187211.250 | 411.793 | $< .001$ |
| $A \times B$ | 16531.250 | 1 | 16531.250 | 36.362 | $< .001$ |
| *Error* $B \times S/A$ | 3637.000 | 8 | 454.625 | | |

- One error term is labelled *Error S/A (Bet-ss)* and has been used to calculate the $F$ ratio for the between-participants factor
- *Error* $B \times S/A$ has been used to calculate the F ratio for every component linked to factor B—the within-participants factor and interaction

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# Simple Main Effects Table For Mixed-Design Stroop Experiment

- There are different approaches to testing simple main effects in mixed designs
- The simplest approach uses **pooled error terms**
- We begin by calculating the between-group variance for each simple main effect
- The calculations are identical to those used for the between-participants design (see Week 7 lecture slides)
- Each pair of simple main effects is tested for significance using the same error term (hence pooled error term approach)

# Simple Main Effects Table For Mixed-Design Stroop Experiment

- The error term for testing the significance of the between-participant effects is the pooled within-group variance for the four cells
- This is calculated identically to a fully between-participants design $\{SS_{S/AB} = [Y] - [AB]; df_{S/AB} = ab(s-1)\}$
- This is used to test the significance of the two simple main effects of the between-participants factor:
  - group at congruent
  - group at incongruent

# Simple Main Effects Table For Mixed-Design Stroop Experiment

| Source | Sum of Squares | Degrees of Freedom | Mean Square | *F* | *P* |
|---|---|---|---|---|---|
| Group at | | | | | |
| congruent | 62.500 | 1 | 62.500 | 0.014 | 0.909 |
| incongruent | 36000.00 | 1 | 36000.00 | 7.938 | 0.023 |
| Error term | 36281.00 | 8 | 4535.125 | | |
| | | | | | |
| Trial type at | | | | | |
| healthy | 46240.000 | 1 | 46240.000 | 101.710 | < .001 |
| schizophrenia | 157502.500 | 1 | 157502.500 | 346.445 | < .001 |
| Error term | 3637.000 | 8 | 454.625 | | |

# Simple Main Effects Table For Mixed-Design Stroop Experiment

- The other error term is the within-participants factor error term from the initial ANOVA (*Error B*$\times$*S/A*)
- This is used to test the two within-participants simple main effects:
  - trial type at healthy
  - trial type at schizophrenia

# Simple Main Effects Table For Mixed-Design Stroop Experiment

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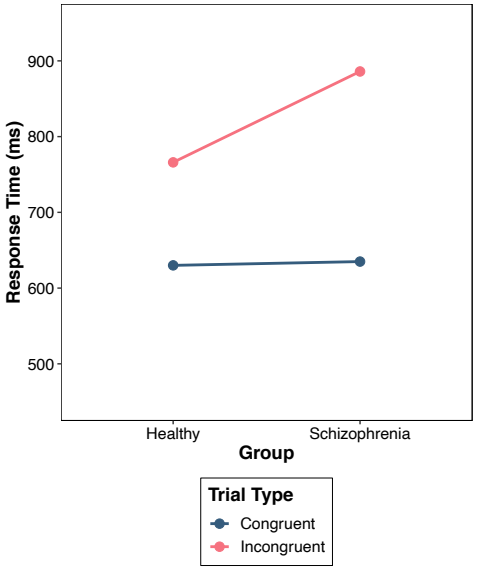
# Simple Main Effects Table For Mixed-Design Stroop Experiment

- An alternative to this approach would be to calculate a separate *t*-test for each pair of means being compared
- We would use independent-samples *t*-tests to test the simple main effects of the between-participants factor
- We would use repeated-measures *t*-tests to test the simple main effects of the within-participants factor

# Simple Main Effects Table For Mixed-Design Stroop Experiment

- Once you have calculated the simple main effects, generate an interaction plot
- Locate the simple main effects in the graph to facilitate interpretation of the interaction

# Interaction Plot



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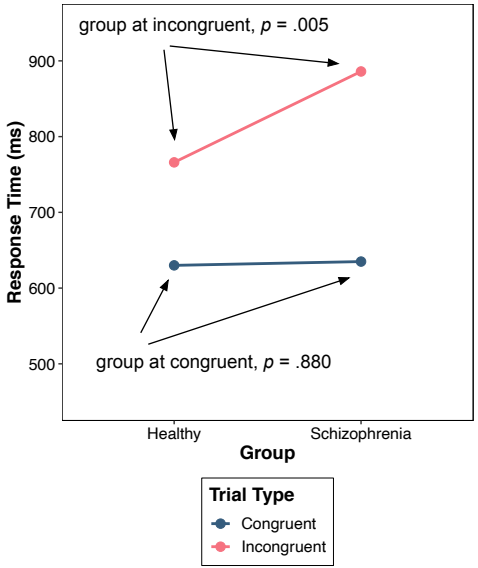

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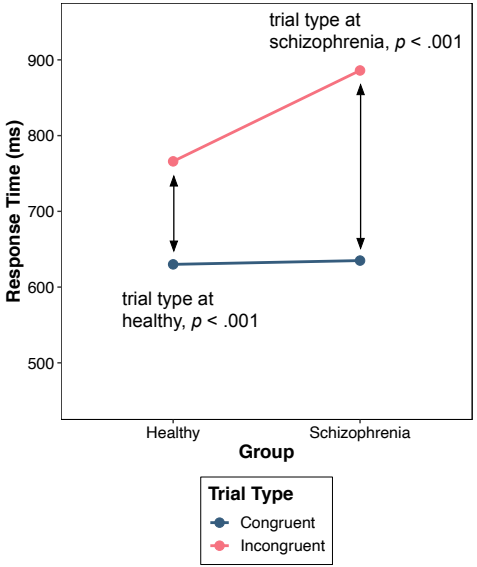

# Simple Main Effects Table For Mixed-Design Stroop Experiment

- Once you have a graph and have calculated the simple main effects, write out the various effects as you were shown in the Week 6 lab session
- This involves reporting the *F* values for each simple main effect and stating the direction of the significant differences
- Once the significant effects have been identified, they must be interpreted
- Write a couple of sentences to describe the nature of the interaction (see the Week 7 lab session for an example)

# What If The Design Has Three or More Levels In Either Factor?

- If the interaction is not significant, any significant main effects for factors with three or more levels will need to be followed up with planned comparisons (*t*-tests) or post-hoc tests (Tukey test)
- When the interaction is significant, the simple main effects for a factor with three or more levels will need to be followed up with planned comparisons or post-hoc tests
- In both circumstances, planned comparisons will often be preferable
- Make sure you use the right type (independent samples *vs.* repeated measures) for the effect you are testing

# Calculating *F* ratios

- The book chapter in the recommended reading includes a demonstration of how to calculate the *F* ratios for a mixed design by hand
- Only study this if you are curious, it is not something you will be assessed upon
- The procedure is very similar to that used when we calculated *F* ratios for a two-factor between-participant design—it uses the same basic ratios (plus one new ratio)

# A Two-Factor Fully Within-Participants Design

- A researcher wants to know if the size of the Stroop effect decreases with practice
- She employs a $2 \times 3$ fully within-participants design:
  - trial type: congruent *vs.* incongruent
  - block: 1 *vs.* 2 *vs.* 3
- Making *trial type* within-participants means we can establish each participant's susceptibility to the Stroop effect
- *block* must necessarily be a within-participants factor, as it requires experience with the task
- There are $2 \times 3 = 6$ conditions; a single group of participants completes each condition

# Hypothetical Data For Fully Within-Participants Design Stroop Experiment

| | $A_1$ congruent | | | $A_2$ incongruent | | |
|---|---|---|---|---|---|---|
| | $B_1$ block 1 | $B_2$ block 2 | $B_3$ block 3 | $B_1$ block 1 | $B_2$ block 2 | $B_3$ block 3 |
| $P_1$ | 700 | 600 | 550 | 910 | 700 | 625 |
| $P_2$ | 600 | 550 | 575 | 850 | 650 | 650 |
| $P_3$ | 480 | 590 | 693 | 720 | 685 | 743 |
| $P_4$ | 630 | 690 | 597 | 830 | 790 | 600 |
| $P_5$ | 720 | 730 | 650 | 845 | 770 | 680 |
| Means | 626.00 | 632.00 | 613.00 | 831.00 | 719.00 | 659.60 |

# ANOVA Table For Fully Within-Participants Design Stroop Experiment

| Source | Sum of Squares | Degrees of Freedom | Mean Square | $F$ | $P$ |
|---|---|---|---|---|---|
| $A$ (trial type) | 95541.63 | 1 | 95541.63 | 68.124 | $< .001$ |
| *Error* $A \times P$ | 5609.87 | 4 | 1402.47 | | |
| $B$ (block) | 42821.60 | 2 | 21410.80 | 2.128 | 0.182 |
| *Error* $B \times P$ | 80503.40 | 8 | 10062.93 | | |
| $A \times B$ | 33872.27 | 2 | 16936.13 | 53.537 | $< .001$ |
| *Error* $A \times B \times P$ | 2530.73 | 8 | 316.34 | | |
| $P$ (participants) | 28847.20 | 4 | 7211.800 | | |

- Each effect has its own error term directly underneath it, which makes locating the degrees of freedom easier

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# Simple Main Effects Table For Fully Within-Participants Design Stroop Experiment

- To test the simple main effects, we calculate the between-group variances as we did in our Week 7 lecture
- The error terms to use are those from the original ANOVA table
- Thus, for the simple main effects of factor $A$ (trial type) at $B_1$, $B_2$, and $B_3$ the error term for testing factor $A$ could be used ($Error\ A \times P$)
- For the simple main effects of factor $B$ (block) at $A_1$ and $A_2$ the error term for testing factor $B$ could be used ($Error\ B \times P$)

# Simple Main Effects Table For Fully Within-Participants Design Stroop Experiment

| Source | Sum of Squares | Degrees of Freedom | Mean Square | $F$ | $P$ |
|---|---|---|---|---|---|
| Trial type at | | | | | |
| block 1 | 105062.50 | 1 | 105062.50 | 74.913 | $< .001$ |
| block 2 | 18922.50 | 1 | 18922.50 | 13.492 | 0.021 |
| block 3 | 5428.90 | 1 | 5428.90 | 3.871 | 0.121 |
| Error term | 5609.87 | 4 | 1402.47 | | |
| | | | | | |
| Block at | | | | | |
| congruent | 943.33 | 2 | 471.67 | 0.047 | 0.954 |
| incongruent | 75750.53 | 2 | 37875.27 | 3.764 | 0.041 |
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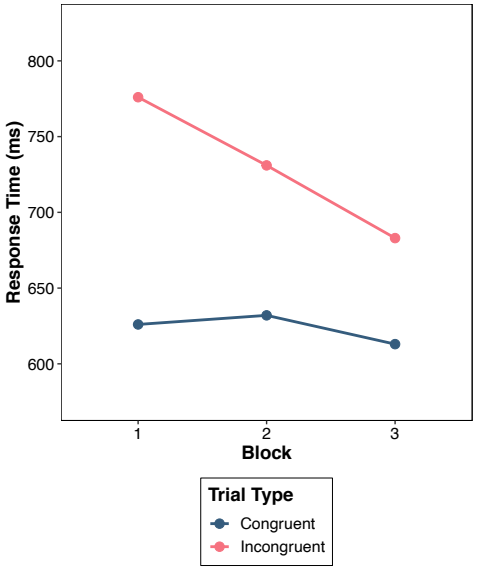

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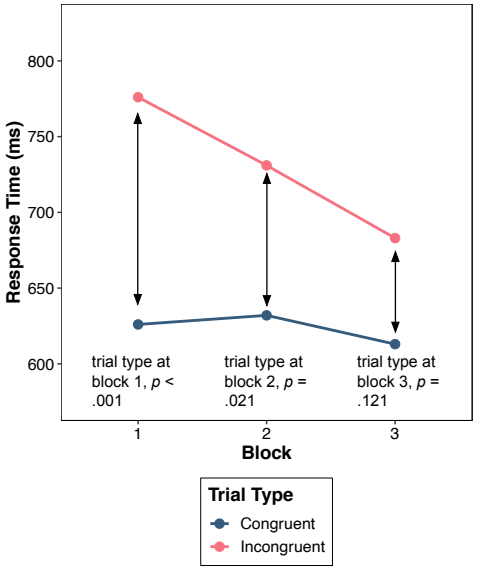

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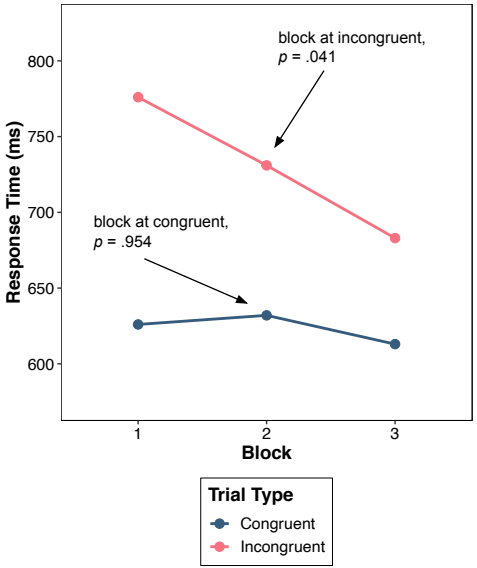
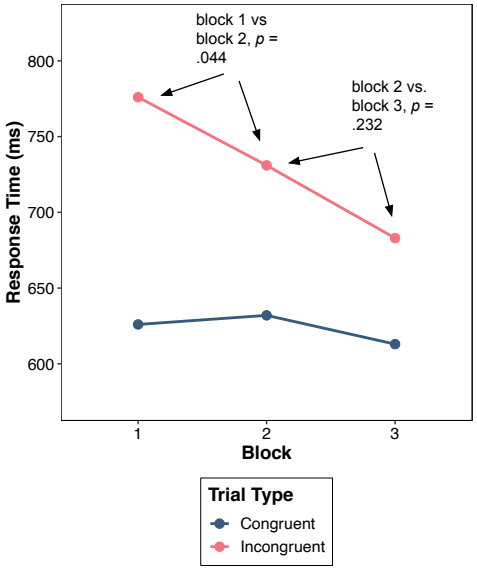


# Follow Up Tests For Simple Main effects Of Factors With Three Or More Levels

- In this instance, one of the simple main effects of our factor with three levels (block at incongruent) was significant
- We therefore need to perform follow up tests (planned comparisons or post-hoc tests) to determine where the differences are located
- I recommend using planned comparisons where possible
- We will evaluate the simple main effect of block at incongruent trials by performing two repeated-measures *t*-tests comparing block 1 *vs.* block 2 and block 2 *vs.* block 3 (i.e., planned comparisons)

# Interaction Plot



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# A Note On The Sphericity Assumption

- The sphericity assumption extends to within-participants factorial designs with factors containing three or more levels
- It also applies to within-participant factors with three or more levels in mixed designs
- R will apply the Greenhouse and Geisser correction if the sphericity assumption is violated
- We'll cover this in more detail in next week's lab

# Additional Resources

- The R code for all plots generated in this lecture (minus annotations) has been uploaded with these slides to the Week 8 lecture folder (R Plots For Lecture 8.R)

# In Next Week's Lab ...

- Running a 2 $\times$ 3 mixed/within-participants ANOVA in R
- Follow-up tests for factors with more than two levels

# References

Roberts, M. J., & Russo, R. (1999, Chapter 11). *A student's guide to Analysis of Variance*. Routledge: London.